# Structured Product Based Variable Annuities*

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**Abstract**

Recently, a new type of variable annuity has been marketed to investors which is based on structured product-like investments instead of the mutual fund-like investments found in traditional variable annuities. Embedding a structured product into a variable annuity introduces substantial complexity into an investment typically considered conservative. In this paper, we describe structured product based variable annuity (spVA) crediting formulas and how they differ from traditional VAs, value the embedded derivative position for a range of example parameters, and calculate the fair cap levels required to fairly compensate investors for the derivative position. We also provide extensive backtests of spVA crediting formulas using our calculated cap levels and compare the results to their underlying indexes. Our findings suggest that the complexity of spVAs can be used to hide fees and reduce the comparability of variable annuities to other investments in the market.

## 1 Introduction

Variable annuities are investments issued by insurance companies and sold through brokers. Traditional variable annuities provide a combination of insurance-like features and the choice of stock and bond portfolios called "subaccounts". Investors in variable annuities can vary their exposure to stock and bond market risk by varying the allocation to the stock and bond subaccounts. The value of a variable annuity changes with changes in the net asset value of the subaccounts an investor has selected, exposing the investor to gains and losses based on market conditions. Distributions on the subaccount investments, that would otherwise be taxable income, accumulates


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tax deferred within deferred variable annuities making this type of investment potentially useful for investors in their peak earning years who have exhausted other tax advantaged investments as they approach retirement.

A new type of variable annuity credits beneficiaries with payoffs similar to those of certain types of structured product rather than those of stock and bond mutual funds. Structured products are derivative debt securities whose payouts are based on the price changes of a reference asset (an index, equity, interest rate, etc.) subject to buffers against losses and caps on gains. The size of the structured products market has increased significantly in the last decade, and is now over $39 billion annually.[1]

The value of structured product based variable annuities (spVAs) is sensitive to price fluctuations of their underlying reference stock and to contract parameters such as the buffer level, cap rate, and term. Structured products underlying these new annuity contracts offer a "buffer" against losses that is financed through a limitation the gains investors can experience at some "cap".

This paper analyzes the risks and benefits of spVAs by looking at three spVAs currently on the market.[2] We provide a detailed comparison the individual product features of the currently-offered spVAs. We decompose the embedded structured product into its component simple assets and highlight the contributions of each component to the overall value of the embedded structured product. We provide backtests of the stated crediting formulas for various caps, buffers, segment lengths, crediting formulas, and fees. We demonstrate that spVAs are much more complex than traditional variable annuities, and that cap levels for most underlying assets and time periods must be quite high to fairly compensate investors for the embedded derivatives position they are sold.

## 2 Currently Available Structured Product Based Variable Annuities

There are currently three structured product based variable annuities available to US investors: AXA Equitable's Structured Capital Strategies Variable Annuity ('SCS', first issued in October 2010), MetLife's Shield Level Selector Single Premium Deferred Annuity ('SLS', first issued in May 2013), and Allianz Life's Index Advantage Variable Annuity ('IA', first prospectus dated August 2013). AXA has sold $2.2 billion of SCS in just the past two years.[3]

[1] Bloomberg Structured Notes Brief, "2012 Review and 2013 Outlook," January 3, 2013.
[2] To the best of our knowledge, the three offerings referenced herein represent the entire spVA universe.
[3] Mercado, Darla. "Structured Products Gain Favor." InvestmentNews, 5 May 2013.

## 2.1 Exposure Types

The defining feature of an spVA is that investors can choose to allocate premiums to "segments" rather than traditional variable annuity subaccounts. Each spVA segments has a defined time period (which we henceforth refer to as the segment's 'term') that determines when the underlying asset is initially observed and when the underlying asset is observed to determine index credits. At the end of an spVA segment, the issuer credits the variable annuity account value with a return based on an underlying asset. The available assets in each of the three spVAs are shown in Table 1 below.

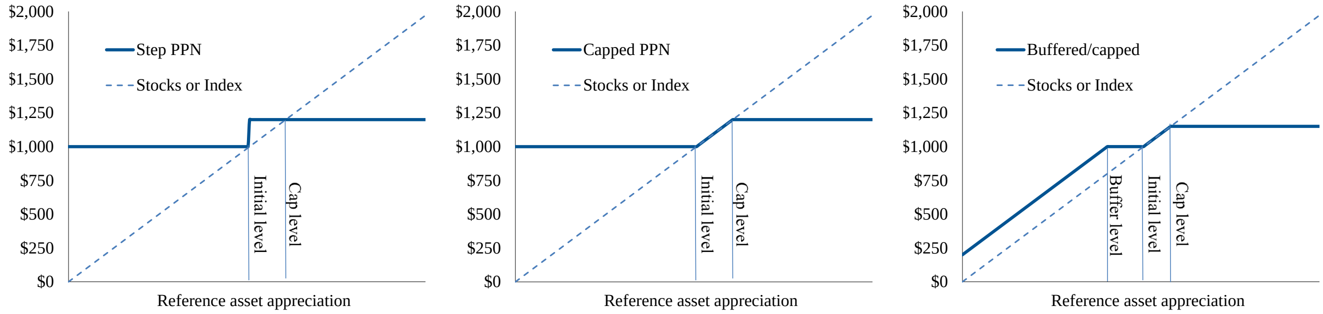
**Table 1:** Currently Available spVA Underlying Assets

| Asset Name | AXA SCS | MetLife SLS | Allianz IA |
|---|---|---|---|
| S&P 500 | ✓ | ✓ | ✓ |
| NASDAQ-100 | ✓ | ✓ | ✓ |
| Russell 2000 | ✓ | ✓ | ✓ |
| MSCI EAFE Index | ✓ | ✓ | ✗ |
| MSCI Emerging Markets Index | ✓ | ✗ | ✗ |
| London Gold Market Fix Price (USD) | ✓ | ✗ | ✗ |
| NYMEX West Texas Int. Crude Oil Front-Month Futures | ✓ | ✗ | ✗ |
| iShares Dow Jones US Real Estate Index Fund | ✓ | ✗ | ✗ |
| Dow Jones-UBS Commodity Index | ✗ | ✓ | ✗ |

To date, spVA crediting formulas have been of two types: a buffered and capped payout, or a principal protected note (PPN) like payout. Both of these embedded structured products can be replicated with option positions, as has been described by a considerable amount of academic work.[4] These difference in maturity payouts is shown in Figure 1.

[4] See (Deng et al., 2009) and (Deng et al., 2012) for details. Our firm has valued over 18,000 structured products including PPNs and buffered/capped notes, and we provide free research reports on each product at `http://slcg.com/products.php`

**Figure 1:** Maturity Payout for a Step PPN (20% step), Capped PPN (20% cap), and Buffered and Capped Product (15% cap, 20% buffer).




(a) Step PPN (b) Capped PPN (c) Buffered and Capped

For all spVA segments, returns are calculated only once at maturity, not periodically over the term of the segment. Also, all payout formulas use price returns, not total returns, effectively excluding any potential income from dividends.

All else being equal, a price index that references securities with a higher dividend yield will degrade over time relative to an index that references with a lower dividend yield. A higher dividend yield effectively decreases the likelihood of positive returns on long call options – and increases the likelihood of negative returns on short put options – referencing the price index.

### 2.1.1 PPN-like Structures

Step PPN spVA segments, as shown in Figure 1(a), pay a fixed amount (the 'step rate') if returns on the underlying asset are positive and otherwise return the amount invested. These products can be valued as a zero coupon bond plus a binary option on the underlying asset. Only two spVAs include Step PPN segments (MetLife's Step Rate option and Allianz's Index Protection Strategy), both of which are only available for the S&P 500 index as of this writing.[5]

MetLife's SLS product also includes a segment with a 100% buffer, which we refer to as the Capped PPN segment shown in Figure 1(b). This segment returns principal paid if the reference asset has a negative return at maturity, and for any positive returns at maturity, pays the return up to a cap. Like the Step PPN structure, this feature is currently only available on the S&P 500 index. We do not analyze PPN-like structures because they appear less common than the structure described below.

[5] Some annuities with similar payout structures are called "Performance Triggered."

### 2.1.2 Buffered and Capped Structures

Other spVA segments include a buffer on losses below a certain threshold (the buffer level) selected by the investor, and a cap on returns above a separate level (the cap) selected by the issuer. As shown in Figure 1(c), this structure exposes the investor to losses beyond the buffer level, such that an investor's maximum possible loss of value on the segment equals 100% minus the buffer level (for a 10% buffer, the maximum loss would be 90%).

Issuers do not disclose the cap level until the segment start date, making it difficult to evaluate a product ex ante. In addition, issuers do not currently publish cap rates, which vary over time and with different buffer levels and underlying assets. In the analysis that follows, we value spVA segments by solving for the cap level that would yield a fair value to investors, as described below.

## 2.2 Withdrawals Before Maturity

Issuers discourage spVA investors from withdrawing their account value by applying a surrender charge (or withdrawal fee). Surrender chrages can be quite large in early years—up to 9% for spVAs—then diminish over time. These charges reduce the amount of an investor's account value that he or she can liquidate at a particular point in time. However, because spVAs are linked to complex derivative positions rather than simple mutual fund-like subaccounts, calculating an account value on any day prior to maturity is complicated and subject to certain assumptions. In this section, we describe how each of the spVAs currently on the market calculate interim account values and surrender charges, which in turn determine what an investor can withdraw at any point in time.

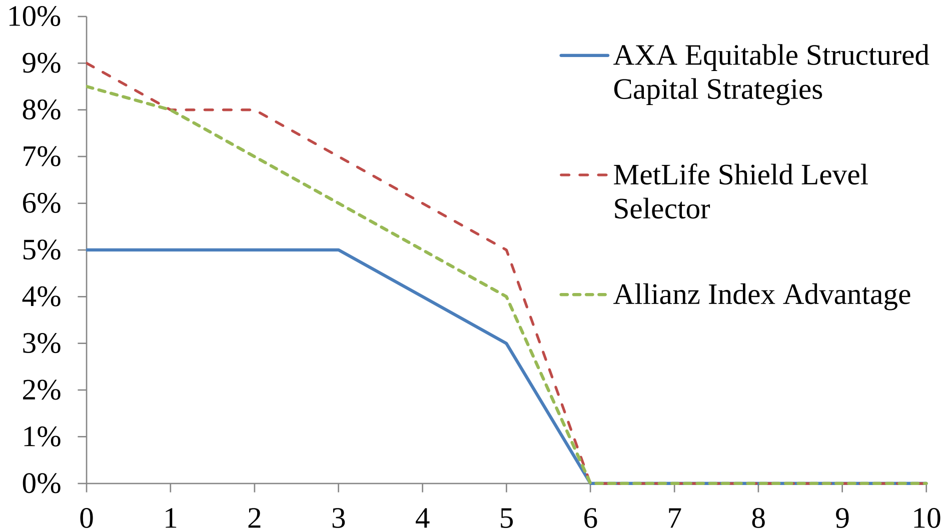
### 2.2.1 Withdrawal Schedule

Variable annuity issuers generally deduct a withdrawal charge when customers request to withdraw funds "to reimburse us for contract sales expenses, including commissions and other distribution, promotion, and acquisition expenses".[6] Figure 2 illustrates the withdrawal schedule for each of the three spVAs as a function of years since contract purchase.[7] The surrender charge changes on policy anniversaries and decreases to zero beyond the sixth policy year for all contracts.

[6] SLS Prospectus, pg. 28.

[7] Withdrawal schedules within a given contract type can vary by state. For the AXA SCS product, we are showing the surrender charge schedule for "Series B" – "Series C" and "Series ADV" have no withdrawal charges.

**Figure 2:** Surrender Charge Schedule for Structured Product Based Variable Annuities




Each contract has a provision that allows policyholders to withdraw a portion of their account value without incurring a withdrawal fee. In the AXA SCS annuity, this provision provides for penalty-free withdrawals up to 10% per year based upon account value. Allianz also includes a maximum 10% annual penalty-free withdrawal, but bases that 10% on premiums paid rather than account value. MetLife bases their 10% penalty-free amount on the most recent policy anniversary's interim value. Withdrawal amounts beyond 10% during the accumulation period of each annuity are charged the surrender charge as depicted in Figure 2.[8]

Allianz applies surrender charges to a "Withdrawal Charge Basis" which consists of premiums paid less non-penalty-free withdrawals and surrender charges – but *not penalty-free withdrawals*. Withdrawals are conducted on a first-in-first-out (FIFO) basis, proportional to the allocation to individual subaccounts (unless otherwise specified by the policyholder). In the case of the MetLife policy, if the interim value has increased above/decreased below the premiums paid, then a policyholder would have access to a larger/smaller dollar amount of penalty-free withdrawals when compared to a similar investment in the Allianz offering.

Another difference between the spVA policies is the net proceeds to a policyholder resulting from a withdrawal. In the case of Allianz and AXA, the net proceeds to the investor is equal to the withdrawal request amount – *i.e.* charges are deducted and applied to the account value. For MetLife, the net proceeds is less than or equal to the withdrawal amount as a result of surrender charges.[9]

[8] For example, a withdrawal amounting to 12% would consist of a 10% penalty-free withdrawal and a surrender charge would be applied to the remaining 2% (assuming no other withdrawals had yet occurred in this policy-year and that there is sufficient funds in the account).

[9] See Allianz IA, pg. 38 and MetLife SLS, pg. 31 for examples of this difference in practice.

### 2.2.2 Interim Account Value Calculation

The spVA issuers determine the value that a policyholder can withdraw at any particular time before maturity using one or both of the following approaches.

The first approach pro-rates the cap and/or buffer level based on the fraction of time that has elapsed in the segment's term since the segment start date. For example, if six months had passed on a 1-year segment, then the cap and/or buffer would be multiplied by 50% and then applied to the returns on the index as of that date. As described by AXA, the pro-rating approach is designed to "discourage owners from withdrawing from a Segment before the Segment Maturity Date where there may have been significant increases in the relevant Index."

The second approach values the underlying options position using option pricing models. The issuer breaks the segment down into its component options, values each option separately, then adds those values (and the value of the embedded zero-coupon bond) to arrive at a total value of the position. Since the spVA segments are based-on European-style options, they can be valued using the well known Black-Scholes options pricing model and current market data. This is nearly identital to the approach we adopt for our valuations in this paper, using the same options decomposition as described by AXA in its SCS prospectus.[10]

Allianz's IA annuity follows the second approach, though they also credit interest respresenting "the value of amortizing the cost of" the option positions over the current policy year. MetLife's SLS uses the first approach, pro-rating both the cap rate and the buffer level. The procedure that AXA uses to determine the interim value is somewhere between these two approaches. AXA compares the calculated value from the pro-rated cap to the value obtained with the second approach and takes the lesser of the two values. Interestingly, AXA does not pro-rate the buffer level, only the cap rate.

## 2.3 An Example Withdrawal Calculation

To illustrate these two approaches, consider for example an investor who purchased an AXA SCS annuity and selected a 3-year segment linked to the NASDAQ-100 starting January 13, 2012 with a buffer level of 10% and a cap level of 20%, into which he invested $100,000. The intrim account value for this segment on June 13, 2013, would be based on two calculations: (1) application of a pro-rated cap (but not buffer), and (2) Black-Scholes valuation of the embedded options contracts as of June 13, 2013.

The first calculation multiplies the fraction of the three year term that had elapsed (50%) by the original cap rate (20%) to get the pro-rated cap (10%). The pro-rated cap value is simply the pro-rated cap rate (10%) times the initial investment ($100,000), or $10,000. Note that this calculation does not take into account either the buffer level or the actual return of the NASDAQ-100 over this period, which was approximately 24.9%.[11]

[10] See Appendix III of the SCS prospectus for details.

[11] The level of the NASDAQ-100 on January 13, 2012 was 2,371.98, and 2,962.90 on June 13, 2013, resulting in a

The second calculation uses current market interest rates, dividend rates, and volatilities to calculate the combined value of the option positions using Black-Scholes. Let's say that combined value was \$115,000. The interim account value for this segment would be the lesser of \$110,000 (\$100,000 initial investment plus \$10,000 pro-rated cap) calculated using the first method and \$115,000 using the second method, or \$110,000.

If the investor wanted to fully withdraw that amount, he would be subject to a 5% surrender charge in accordance with the schedule in Figure 2. However, he would also be able to withdraw up to 10% of his interim account value without incurring this fee. Therefore, on June 13, 2013, he could withdraw \$11,000 penalty-free (\$110,000 times 10%), and pay \$4,950 in surrender charges (\$99,000 times 5%). His total liquidation proceeds would then be \$110,000 minus \$4,950 or \$105,050.[12]

The pro-rated cap is effectively a ceiling on any potentially high values determined by the derivatives valuation. Also, AXA notes that interim account values may be lower than the initial investment, even if the underlying index has experienced a positive return since the date of initial investment.

## 2.4 Annual Expenses

Allianz applies a 1.25% annual fee to the "withdrawal charge basis". This account reflects the premiums paid and non-penalty-free withdrawal and surrender charges. AXA applies an annual contract fee ranging from 0.65% for "Series ADV" contracts to 1.65% for the "Series C" contracts based upon the daily net assets. MetLife does not specify an annual contract fee, but state that sales representatives make 5% − 6% upfront, or can opt for a smaller upfront sales commission in exchange for annual concessions of between 0.6% − 1% of the account value. As a result, we can estimate the contract fee to cover commissions is at least 1% per annum.

## 2.5 Volatility

The indexes available for spVAs have very different volatility characteristics. In Figure 3 we plot the average 12 month implied volatility of the underlying indexes available in spVAs.[13] For most of the period from 2008 − 2013, the S&P 500 was the least volatile of these indexes. Therefore, S&P 500 linked spVA segments tend to be valued higher (having a lower fair cap level) than those linked to other indexes.[14]

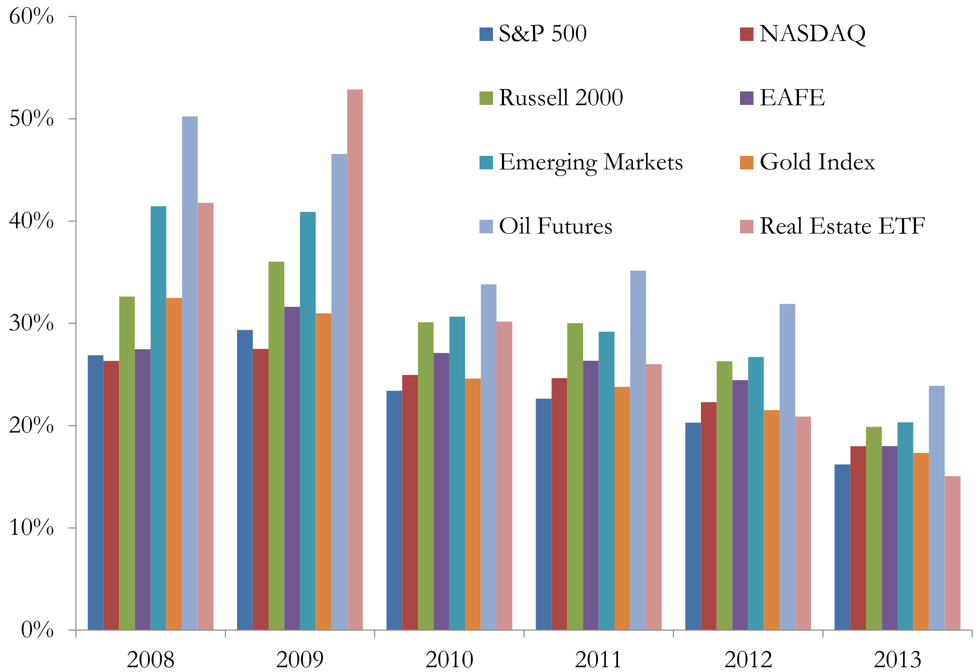
return of 24.9%.

[12] AXA's SCS prospectus notes that "when a contract is surrendered in certain states, the free withdrawal amount is not taken into account when calculating the amount of the withdrawal."

[13] As noted below, we use the 12 month implied volatility of an exchange-traded fund linked to the underlying asset if volatility data is not available for the asset itself.

[14] We choose to focus on implied volatility here because implied volatility is the parameter used to determine the value of the underlying option positions in the Black-Scholes model.

**Figure 3:** Average One-Year Implied Volatility for spVA Underlying Assets



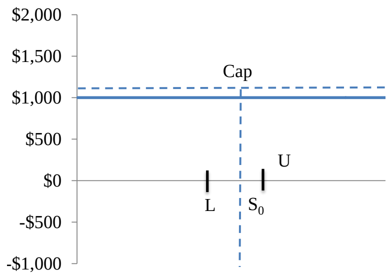

# 3 Valuing the Embedded Derivative Position

The structured products underlying variable annuities are buffered against downside losses and cap large positive returns, putting them in a class of structured products branded variously as Buffered Note, Buffered Index-Linked Note, or Buffered Equity Note, amongst others, depending on the issuer. In essence, these products offer the buyer protection against small losses in exchange for capping potential gains.

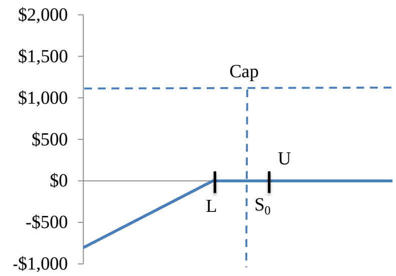
## 3.1 Decomposition

The payoffs underlying buffered and capped structured products can be valued using a variety of techniques, but for illustrative purposes, we here use a decomposition approach. Buffered and capped structured products can be decomposed into a portfolio of four assets: (1) a zero coupon bond, (2) a short European put option with a strike price equal to the buffer level, (3) a long European at-the-money call option, and (4) a short European call option at the cap level. Figure 4 gives a graphical depiction of this decomposition.

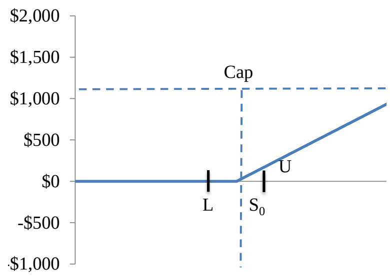
**Figure 4:** Maturity Payoff of Option Decomposition of a Buffered and Capped Structured Product Example



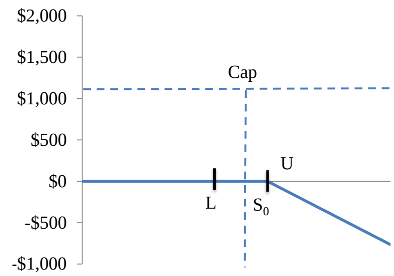

(a) Long Zero Coupon Bond



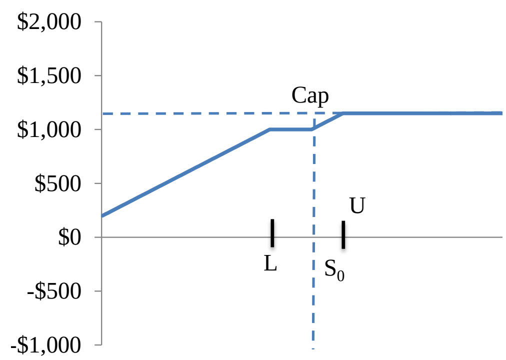

(b) Short OTM Put




(c) Long ATM Call



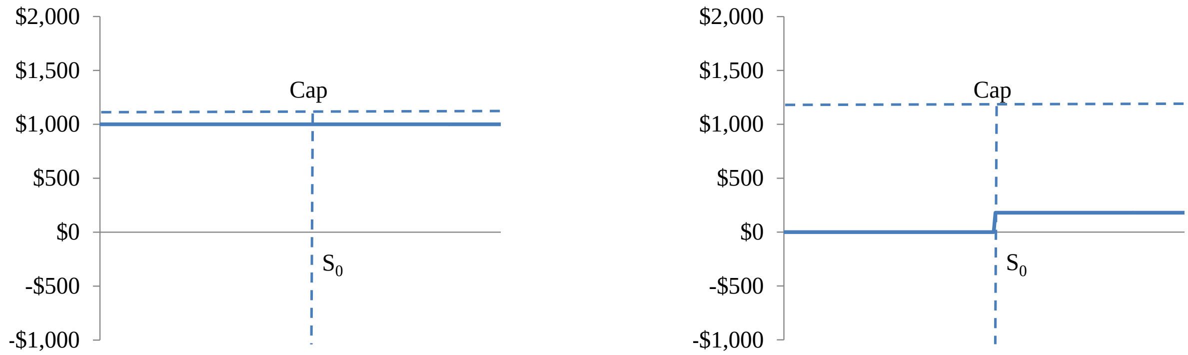

(d) Short OTM Call




(e) Buffered and Capped Payoff

The payoff to a capped PPN product can be decomposed using the same decomposition as in Figure 4, but without the short out-of-the-money put option. The decomposition of step-PPN structured products requires a (1) zero coupon bond and (2) a long at-the-money cash-or-nothing binary call option. Figure 5 illustrates this decomposition.

**Figure 5:** Maturity Payoff of Option Decomposition of a Step-PPN Structured Product Example




(a) Long Zero Coupon Bond

(b) Long Cash-or-Nothing ATM Call

With the decomposition approach each component can be valued independently within an appropriate model and then the values of the components can be summed to compute the overall product value. Since the options in the decomposition are European, each can be priced using the closed-form formulas of the Black-Scholes model for example. Using this model, the annuity can be valued simply by knowing the implied volatility and dividend yield of the reference asset, as well as the spVA segment's time to maturity, cap level, and buffer level.

Abstracting to the mathematical representation of the decomposition, we develop the closed-form valuation formula for this type of structured product. Consider a timeline with $n$ segments of any arbitrary duration such that $t_0 < t_1 < \ldots < t_n$. The present value of the investment equals the product of the returns over each time period, discounted to present value:

$$PV = I_0 e^{-r(t_n - t_0)} \mathbb{E}\left[\prod_{i=1}^{n}(1+R_i)\right] = I_0 e^{-r(t_n - t_0)} \prod_{i=1}^{n} \mathbb{E}\left[1+R_i\right] \tag{1}$$

where $I_0$ is the initial account value, $r$ is the riskless rate over the period, and $R_i$ is the return for the $i^{\text{th}}$ segment. We simplify Equation 1 using the fact that returns are independent and identically distributed. The return of the product over any individual segment is simply the sum of the value of each of its components.

Using the Black-Scholes model, we have the following equation for the expected return for a buffered and capped product

$$\begin{aligned} \mathbb{E}\left[1+R_i\right] &= 1 - \left(\frac{FV_{\text{Put}}\left(S_0, K_b, \tau, r, q, \sigma\right)}{S_0}\right) \\ &+ \left(\frac{FV_{\text{Call}}\left(S_0, K = S_0, \tau, r, q, \sigma\right)}{S_0}\right) - \left(\frac{FV_{\text{Call}}\left(S_0, K_c, \tau, r, q, \sigma\right)}{S_0}\right) \end{aligned} \tag{2}$$

where $q$ is the dividend yield of the index/stock with initial value $S_0$ and implied volatility $\sigma$. $\tau$ is the length of the particular segment and the strike prices are defined by

$$\begin{aligned} K_b &= S_0(1 - \text{Buffer}) \quad (3)\\ K_c &= S_0(1 + \text{Cap}). \quad (4) \end{aligned}$$

For a capped-PPN, the following equation gives the expected return is

$$\mathbb{E}\left[1 + R_i\right] = 1 + \left(\frac{FV_{\text{Call}}\left(S_0, K = S_0, \tau, r, q, \sigma\right)}{S_0}\right) - \left(\frac{FV_{\text{Call}}\left(S_0, K_c, \tau, r, q, \sigma\right)}{S_0}\right) \quad (5)$$

while for a step-PPN the expected return is

$$\mathbb{E}\left[1 + R_i\right] = 1 + \left(\frac{FV_{\text{Binary Call}}\left(S_0, K = S_0, \tau, r, q, \sigma\right)}{S_0}\right) \quad (6)$$

The formulas for the price of options in the Black-Scholes model are

$$\begin{aligned} FV_{\text{Call}}\left(S_0, K, \tau, r, q, \sigma\right) &= FN(d_1) - KN(d_2) \quad (7)\\ FV_{\text{Put}}\left(S_0, K, \tau, r, q, \sigma\right) &= KN(-d_2) - FN(-d_1) \quad (8)\\ FV_{\text{Binary Call}}\left(S_0, K, \tau, r, q, \sigma\right) &= KN(d_2) \quad (9) \end{aligned}$$

where

$$d_1 = \frac{\ln\left(\frac{F}{K}\right) + \frac{\sigma^2\tau}{2}}{\sqrt{\sigma^2\tau}}, \quad d_2 = d_1 - \sqrt{\sigma^2\tau}, \quad \text{and} \quad F = e^{(r-q)\tau}. \quad (10)$$

Given the implied volatility and dividend yield of the underlying asset as well as the terms of the annuity contract, the value of the embedded structured product can be computed exactly in this model using Equation 1. Any other empirically consistent model can be applied to Equation 1 to value the structured product.[15]

## 3.2 Credit Risk and Other Features

While credit risk is small for very short term structured products and not incorporated into valuation models, the financial crisis of 2008–2009 demonstrates that even large financial institutions can fail; indeed, Lehman Brothers was one of the largest issuers of structured products, and the individuals who purchased their products faced tremendous losses when Lehman Brothers went bankrupt. In our valuations of structured products, we incorporate the CDS rate of the issuer

[15] According to the AXA SCS prospectus, it uses an indentical method for valuing spVA segments for the purposes of calculating an interim value before maturity.

in order to quantify the risk that they will not meet their debt obligations. This corresponds to methods outlined in the vulnerable options literature. See (Hull, 2011), (Hull and White, 1995) and (Jarrow and Turnbull, 1995).[16]

Even without embedded structured product payoffs, variable annuity contracts can be complex investments. They often have a variety of benefits and penalties which make them difficult to value, such as death benefits, surrender charges, and so on. These features have been the subject of much academic work and have been valued using risk-neutral option pricing theory, stochastic control models, and Monte Carlo simulations.[17]

# 4 Sensitivity Analysis

## 4.1 Fair Cap Levels

In order to value an spVA segment, we must know (1) the term remaining on the segment, (2) the risk-free rate for that term, (3) the underlying asset, (4) implied volatility and dividend yield of the underlying asset, (5) the cap and buffer level of the segment. All of these can be found in the terms of the prospectus or by market data except for the cap level, which is set by the issuer. Therefore, in our valuations we either assume a cap level or we solve for a cap level that would make the value of the segment equal to the initial premium paid (we define this cap level to be the *fair cap level*). In the latter case, the higher the fair cap level, the lower the value of the underlying segment.

Including the effect of upfront fees or sales commissions on the product is equivalent to solving for the cap level that results in a value that is equal to the premium paid less the commissions. In the analysis that follows, we assume that the issuer charges a 1% premium by offering a lower cap than would fully compensate investors for the embedded option positions. Thus, our reported 'fair cap levels' are the caps that yield an combined option and bond position worth 99% of premiums paid. We discuss the effects of this level in Appendix A below.

We have solved for the fair cap level on each monthly date between January 13, 2005 and April 13, 2013 for each of the underlying indexes and each of the terms and buffer levels available in the AXA SCS product.[18] The results for 1 year terms are shown in Figure 6 below for the S&P 500,

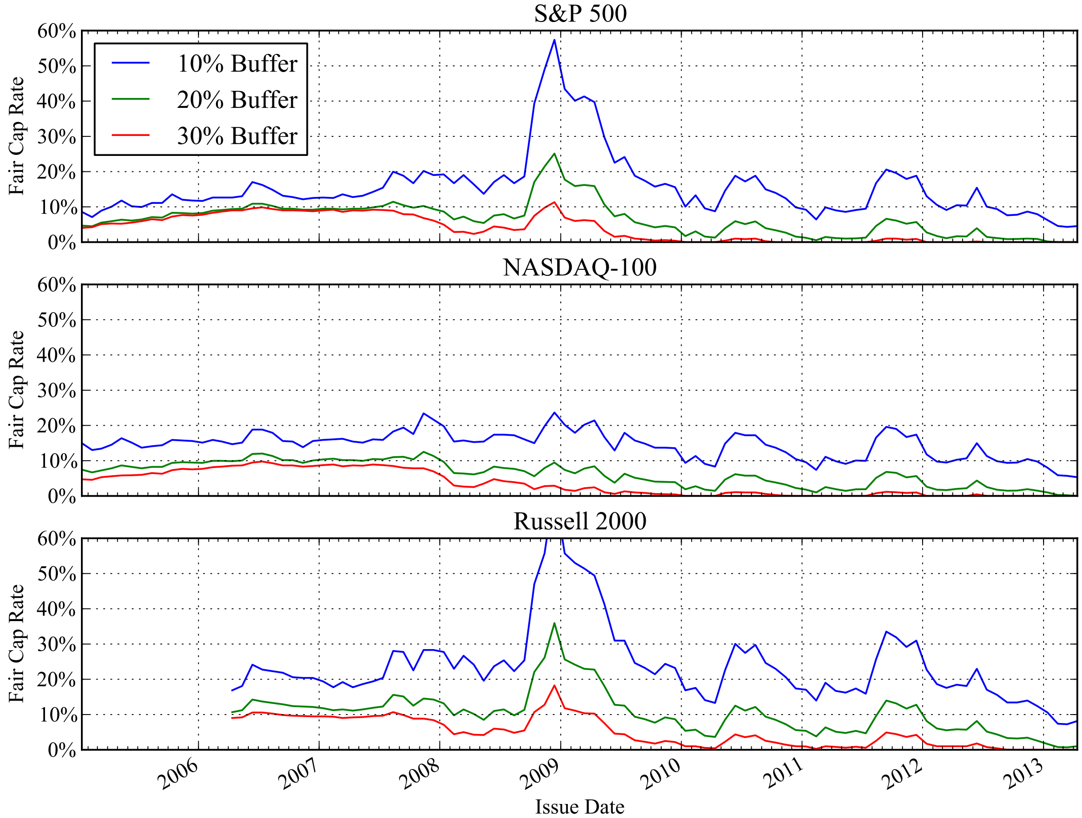
[16] From (Hull, 2011), page 533: "...any derivative promising a payoff at time $T$ can be valued by increasing the discount rate that is applied to the expected payoff in a risk-neutral world from the risk-free rate $y$ to the risky rate $y^*$." In our formulation above, this would involve changing $r$ from a risk-free rate to a risky rate, perhaps by adding the CDS rate of the issuer.

[17] See (Milevsky and Posner, 2001), (Bauer et al., 2008) and (Dai et al., 2008). For a basic introduction, see (Blamont and Sagoo, 2009).

[18] We perform this calculation using custom valuation software developed in Python. Market data is collected from Bloomberg, LLC. We match the volatility to the term of the segment if possible, otherwise we use the longest dated volatility available. If no implied volatility is available for the asset, we use the implied volatility of an exchange-traded fund (ETF) that tracks the asset.

NASDAQ-100, and Russell 2000 indexes.[19]

**Figure 6:** Fair Cap Levels Over Time
*One year term*




Around the financial crisis, the implied volatility for one year options on the S&P 500 and the Russell 2000 increased significantly. As a result, the fair cap level in Figure 6 increases to compensate investors for their increased exposure to loss. The increase is less significant for the NASDAQ-100 since the stocks in this index only have indirect exposure to the financial industry.

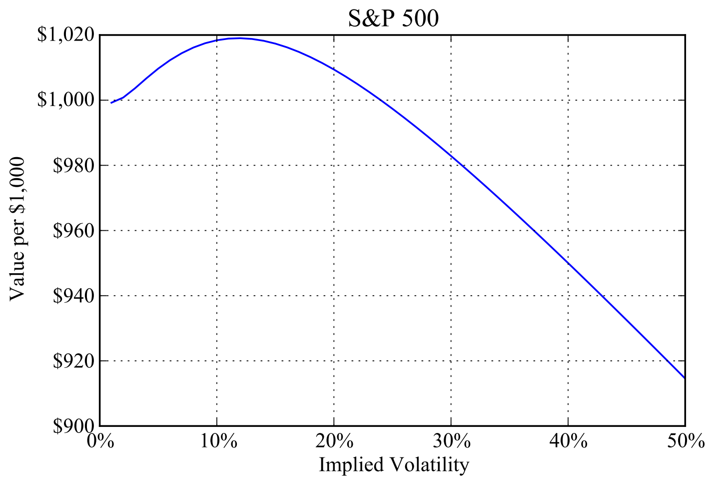
We find that the fair cap increases dramatically as the buffer is relaxed. For example, around the financial crisis the insurance company would have had to structure a 10%-buffer one-year product with a cap of more than 60% to be equivalent to a 20%-buffer one-year product with a cap of approximately 30%. At this time investors were exposed to a higher probability of large losses over this short time span and would have required significant compensation (in the way of a higher cap) to compensate for a smaller buffer.

[19] Bloomberg's volatility data for the Russell 2000 begins in early 2006.

## 4.2 Effect of Implied Volatility

The value of buffered and capped structured products is sensitive to the implied volatility of the underlying asset.[20] In particular, the value of the product goes down the higher the implied volatility of the underlying asset. Figure 7 shows the change in value of an S&P 500 linked, 1 year segment with a buffer of 10% and a cap of 20%, valued as of April 13, 2013.

**Figure 7:** Value of 1 Year spVA Segment Linked to the S&P 500 with 10% Buffer, 20% Cap, as of April 13, 2013

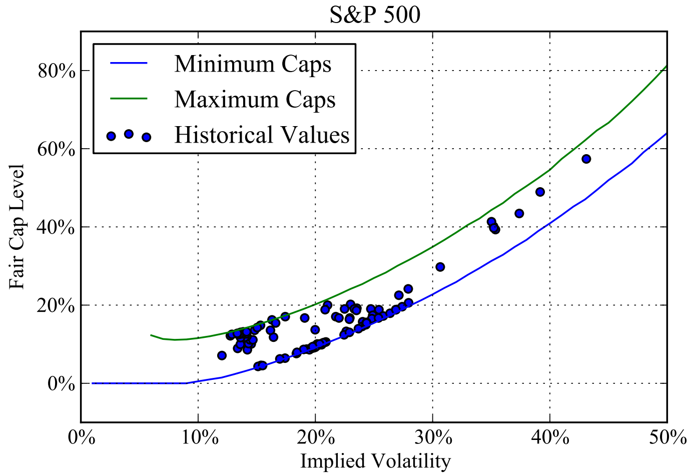



In Figure 8, we plot the fair cap levels for one-year, S&P 500 linked segments valued on each month in our time period, assuming a buffer level of 10% against the implied volatility of the S&P 500 on that day. We also plot the hypothetical maximum and minimum fair caps for each level of implied volatility across the entire time period.[21] These valuations demonstrate that higher levels of implied volatility require higher fair caps in order to compensate investors for reduced option value.

[20] An option position's price sensitivity to implied volatility is known as vega ($\nu$), or sometimes as kappa ($\kappa$).

[21] The hypothetical maximum and minimum fair caps are calculated by calculating the fair cap at every implied volatility in the displayed range on each monthly date in our time period. Each date will differ in its risk free rate and dividend yield.

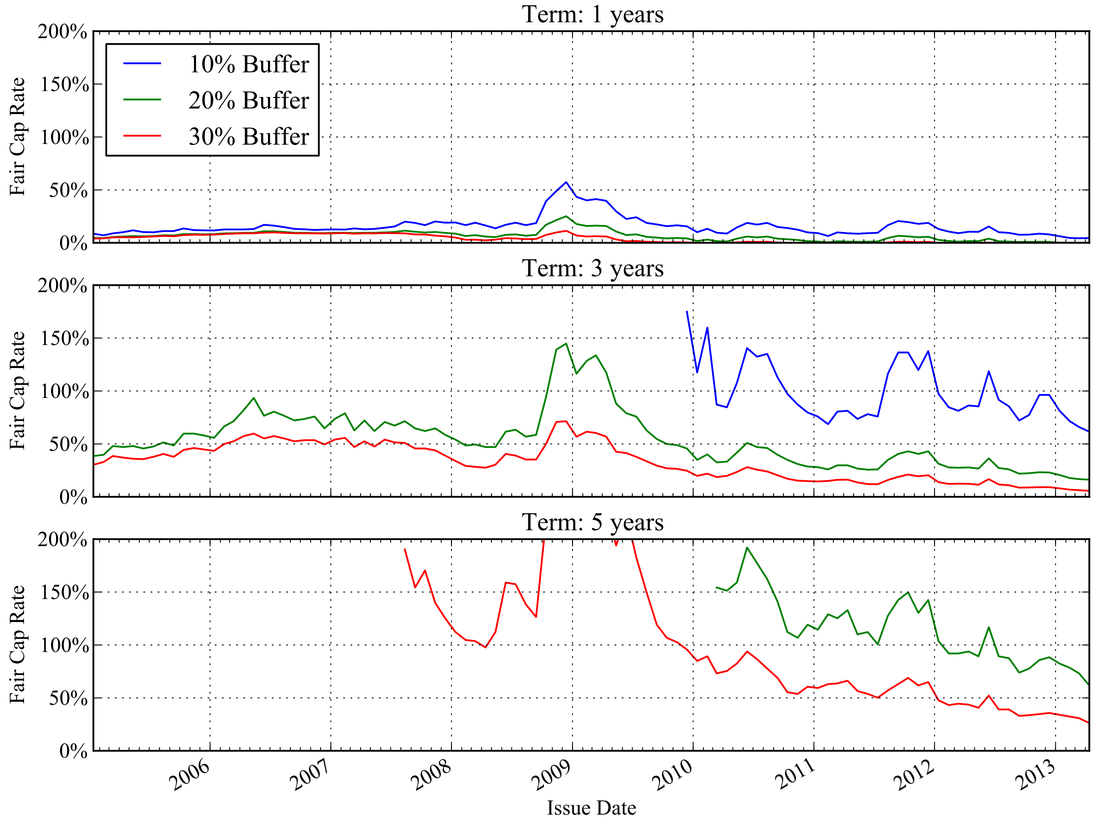
**Figure 8:** Implied Caps on a One-Year spVA Segment
Linked to the S&P 500 with 10% Buffer




## 4.3 Effect of the Term

The value of an spVA segment also depends on time to maturity. The buffered and capped options position tends to be worth less with longer times to maturity, requiring higher cap levels. Figure 9 shows the fair cap levels for three terms (one year, three year and five year) for the same underlying index – the S&P 500. Comparing the three figures, all on the same scale, it is clear that for the for the longest terms investors should require cap levels that are literally off the charts. The fact that spVA segments tend to be worth less with longer time periods (keeping the cap constant) may explain why issuers tend to offer high buffer levels only on longer-term products.

**Figure 9:** Fair Cap Levels Over Time

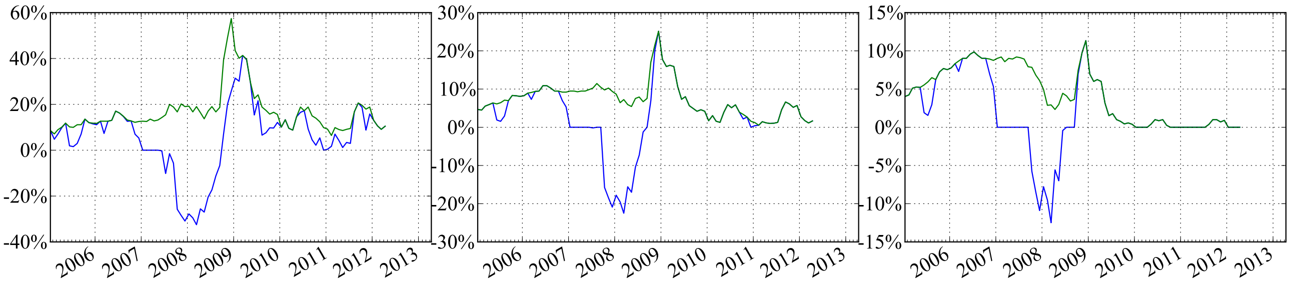



# 5 Backtesting Crediting Formula

Since structured product based variable annuities are a relatively new entry into the insurance industry, there exists no historical data on the cap levels applied to contracts. To make progress on the historical backtests, we use the fair cap levels of Section 4 that include a 1% upfront fee and observe the payout investors would have received at the end of each segment.

In Figure 10 we show the realized return over the segment for each month in our sample period.

**Figure 10:** Backtest of a One-Year spVA Segment Linked to the S&P 500
*Blue line is backtested return, green line is fair cap level*




(a) 10% Buffer (b) 20% Buffer (c) 30% Buffer

In Table 2, we present summary statistics for the historical backtest of the segment performance. The second and third columns of Table 2 give the average *price return* and the standard deviation of these price returns for each index over the periods that are simultaneous to the segments studies in the right four columns.

**Table 2:** Summary Statistics for Historical Backtest of One-Year Segment spVAs with a 1% Upfront Fee

| Index | Index Average Return | Standard Deviation | Buffer Level | Count | Average Return | Standard Deviation | Percent Capped |
|---|---|---|---|---|---|---|---|
| S&P 500 | 4.4% | 18.8% | 10% | 88 | 5.7% | 14.7% | 27.3% |
| | | | 20% | 88 | 2.9% | 8.6% | 63.6% |
| | | | 30% | 88 | 1.8% | 4.7% | 68.2% |
| NASDAQ-100 | 10.0% | 21.0% | 10% | 88 | 6.7% | 12.3% | 44.3% |
| | | | 20% | 88 | 2.8% | 6.6% | 64.8% |
| | | | 30% | 88 | 1.7% | 3.8% | 65.9% |
| Russell 2000 | 6.2% | 21.4% | 10% | 73 | 6.9% | 17.9% | 13.7% |
| | | | 20% | 73 | 4.4% | 10.3% | 42.5% |
| | | | 30% | 73 | 2.4% | 5.1% | 56.2% |

Our historical backtests show that the crediting formulas of the spVA segments reduce the volatility of realized returns compared to their underlying index returns. For example, the one-year spVA linked to the S&P 500 with 10% buffer decreases the standard deviation of returns by about 21.8% (14.7%/18.8% − 1). Since the cap is not very restrictive at this buffer level, it is not surprising that the 10% segment for each index out-performs the respective index on average. The one-year S&P 500 spVA with a 30% buffer has a standard deviation that is 75% that of the underlying index.

For a 10% buffer, the fair cap level is high and as a result few of the realized returns (27.3%) are equal to the fair cap level – see Figure 10(a). On the other hand, for a buffer of 30%, the fair cap level is low and many of the realized returns (68.2%) are equal to the fair cap level – see Figure 10(c). These statistics are summarized in the last column of Table 2.

# 6 Discussion

Annuities have evolved into enormously complex products, and while often marketed as conservative investments for retirement savings purposes, they carry significant risks. In this paper, we analyzed the risks and benefits of structured product based variable annuities (spVAs) by looking at three spVAs currently on the market. We provided a detailed comparison the individual product features of the currently-offered spVAs and decomposed the embedded structured product into its component simple assets. We provided backtests of the stated crediting formulas for various caps, buffers, segment lengths, and underlying assets. We demonstrated that spVAs are much more complex than traditional variable annuities, and that the principal protection offered by the buffer may not compensate for the expected value lost due to the cap on potential upside gains.

Unlike traditional variable annuities, spVAs yield the issuer a tremendous degree of pricing control through the cap. Our analysis suggests that fair cap levels are often very high; any lower cap would imply a premium charged to the investor. While traditional variable annuities must recoup high sales commissions through charges and penalties, spVA issuers can recoup those expenses by setting the cap. This may explain why the three spVAs currently in the market differ in regards to their surrender charge schedules. An issuer could offer a product with a lower surrender charge or lower annual fee but select lower caps on its spVA segments to maintain profitability.[22]

Our results suggest that this class of variable annuity is a markedly different type of investment than traditional variable annuities, and requires a much more sophisticated analysis of the product parameters, especially the tradeoff between capped upside potential and buffered downside losses. To many investors, the term annuity suggests stability, low risk, and guaranteed income. While spVAs still bear this title, their crediting formulas are highly complex and the resulting account accumulation may be very different than what investors expect. Due to this complexity, it will like prove difficult for investors to compare spVAs to each other or to traditional variable annuities, which allow an investor to select the degree of equity exposure desired, rather than allowing the risk and return of his or her investment to be determined by the issuer of the product.

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# A Effect of Fees

We use this appendix to show the effect of fees on the calculated fair cap level. In Table 3, we show average return, standard deviation and percent capped similar to Table 2 but for different fee levels. This additional historical backtest shows the sensitivity of the distribution of realized returns to the level of embedded fees.

**Table 3:** Summary Statistics for Historical Backtest of One-Year Segment spVAs

| Fee | Buffer Level | Count | Average Return ($\bar{r}$) | Standard Deviation ( $\sigma_r$) | Percent Capped |
|---|---|---|---|---|---|
| **S&P 500** ($\bar{r} = 4.4\%$, $\sigma_r = 18.8\%$) | | | | | |
| 0.00% | 10% | 88 | 6.4% | 15.5% | 4.5% |
| 0.50% | | | 6.2% | 15.1% | 12.5% |
| 1.00% | | | 5.7% | 14.7% | 27.3% |
| 1.50% | | | 4.9% | 14.1% | 37.5% |
| 0.00% | 20% | 88 | 4.7% | 9.6% | 45.5% |
| 0.50% | | | 3.8% | 9.0% | 55.7% |
| 1.00% | | | 2.9% | 8.6% | 63.6% |
| 1.50% | | | 2.0% | 8.1% | 65.9% |
| 0.00% | 30% | 88 | 3.4% | 5.7% | 59.1% |
| 0.50% | | | 2.5% | 5.2% | 64.8% |
| 1.00% | | | 1.8% | 4.7% | 68.2% |
| 1.50% | | | 1.3% | 4.3% | 70.5% |
| **NASDAQ-100** ($\bar{r} = 10.0\%$, $\sigma_r = 21.0\%$) | | | | | |
| 0.00% | 10% | 88 | 8.4% | 13.7% | 31.8% |
| 0.50% | | | 7.6% | 13.0% | 38.6% |
| 1.00% | | | 6.7% | 12.3% | 44.3% |
| 1.50% | | | 5.8% | 11.7% | 50.0% |
| 0.00% | 20% | 88 | 4.6% | 7.5% | 58.0% |
| 0.50% | | | 3.7% | 7.0% | 61.4% |
| 1.00% | | | 2.8% | 6.6% | 64.8% |
| 1.50% | | | 2.0% | 6.2% | 64.8% |
| 0.00% | 30% | 88 | 3.1% | 4.4% | 64.8% |
| 0.50% | | | 2.3% | 4.1% | 64.8% |
| 1.00% | | | 1.7% | 3.8% | 65.9% |
| 1.50% | | | 1.2% | 3.5% | 67.1% |
| **Russell 2000** ($\bar{r} = 6.2\%$, $\sigma_r = 21.4\%$) | | | | | |
| 0.00% | 10% | 73 | 7.5% | 18.9% | 5.7% |
| 0.50% | | | 7.2% | 18.4% | 9.6% |
| 1.00% | | | 6.9% | 17.9% | 13.7% |
| 1.50% | | | 6.4% | 17.2% | 19.2% |
| 0.00% | 20% | 73 | 5.8% | 11.5% | 29.5% |
| 0.50% | | | 5.1% | 10.9% | 38.6% |
| 1.00% | | | 4.4% | 10.3% | 42.5% |
| 1.50% | | | 3.7% | 9.8% | 42.5% |
| 0.00% | 30% | 73 | 3.8% | 6.1% | 39.8% |
| 0.50% | | | 3.1% | 5.6% | 52.1% |
| 1.00% | | | 2.4% | 5.1% | 56.2% |
| 1.50% | | | 1.7% | 4.7% | 64.4% |